Motion Planning and Control of Two Quadcopters with Cable-suspended Point Mass Payload

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Abstract. Considering the low payload carrying capacity for a single quadcopter, one option to increase the payload carrying capacity is the use of more than one quadcopter. This work focuses on trajectory planning and control of two quadcopters with a cable-suspended point mass payload system using a leader-follower scheme. For safe and stable transportation suspended payload, the quadcopters should be controlled to not lead to cable slackness. Accordingly, wrench closure analysis is carried out for the follower quadcopter with respect to the leader quadcopter, which helps design desired trajectory generation for the quadcopters. The performance of the proposed motion planning strategy is verified by conducting simulation in SIMSCAPE multibody package in MATLAB.

Keywords: Aerial transportation, Multiple quadcopters, Cable-suspended payload, Wrench closure workspace

1 Introduction

Quadcopters have a wide range of applications both in the military and civil sector, because of which they have been a subject of research for many research groups. Some of the applications are aerial photography, agriculture use, aerial surveillance, humanitarian operations in disaster situations, etc. In many of these applications, the quadcopter is required to carry some specific type of payload.

Various methodologies are presented in the literature to transport payloads from one place to another. For example using robotic arms/grippers [1–3] or by suspending the payload using cables [4–11] or direct attachment to the chassis [12,13]. However, the agility of the quadcopter retains while transporting payload by suspending it through cables because it decouples the attitude dynamics of the quadcopters. To increase the payload carrying capacity, one has to increase the capacity of motors and need to change the overall structure of the chassis. On the other hand, another alternative to increase the payload carrying capacity is by using multiple quadcopters.

A motion planning approach using a transition-based rapidly-exploring random tree algorithm was implemented in [14] for the system consisting of three aerial robots transporting a rigid payload to ensure the wrench-feasibility constraint in the cables. A wrench set analysis technique from the cable-driven parallel robots (CDPR) was implemented in [15] for aerial cable towed system (ACTS) consisting of the quadcopters which can be used for the cable-tension distribution and motion planning. In [16], cooperative transportation of a cable-suspended payload using multiple quadcopters was demonstrated in which reconfigurable parallel robot techniques were used to avoid specifying the prior forces in the cables and get rid of the tension distribution algorithms. Geometric non-linear control for the multiple quadcopters with cable-suspended payload system was implemented in the [17] by simulations and experiments. All of these methods are subjected to higher computational costs to run trajectory planning and control algorithms. Moreover, some methods can not be directly implementable in outdoor environment settings.

The present work focuses on trajectory planning and control for cable-suspended point mass payload using two quadcopters. The governing dynamical equation of motion of the system is derived using the Newton-Euler equation, considering cable remains taut during transportation. A leader-follower scheme is considered in which the leader quadcopter is autonomously controlled to direct the motion of the entire system, and the follower quadcopter is commanded for safe and stable transportation of the payload while keeping positive tension in the cables. Accordingly, wrench closure workspace analysis is carried out for the follower quadcopter, ensuring positive tension in the cables. This workspace analysis is further extended for trajectory generation for the follower quadcopter, which results in tracking the prescribed trajectory of the payload. Finally, the SIMSCAPE multibody dynamic toolbox of SIMULINK (in MATLAB) is used to check the performance of the developed motion planning and control strategy.

2 Method

The line diagram of two quadcopters with a cable-suspended point mass payload system is shown in Fig. 1. The inertial frame of reference, $\{I\}$, is represented as three orthonormal vectors $[a_1,\ a_2,\ a_3]^T$ where a_3 is taken in the vertically upward direction. Body frame of reference to each quadcopter $\{B_i\}$, where $i = \{1, 2\}$ is considered using three orthogonal unit vectors, $[b_{1_i},\ b_{2_i},\ b_{3_i}]^T$ where $i = \{1, 2\}$, attached to its center of gravity (CG) where first body axis, i.e., b_{1_i}, is pointed towards the middle of first and second motors as shown in the Fig. 1 and third axis, i.e., b_{3_i}, is pointed perpendicular to plane of the quadcopter. The mass and moment of inertia about CG of each quadcopter of i^{th} quadcopter is denoted as m_i, J_i. It is assumed that point mass payload, m_0, is suspended using inextensible cables to the quadcopter's CG.

The translational position of the payload is denoted as $X_0 \in \mathbb{R}^3$. The length of the cable from the quadcopter 1 and 2 to payload is denoted as l_1 and l_2 respectively. The translational position and attitude of the quadcopter in frame $\{I\}$ is denoted as $X_i = [x_i, y_i, z_i]^T \in \mathbb{R}^3$ and $R_i \in SO(3)$ respectively. Standard ZXY Euler angle parameterization is used to represent the attitude of the

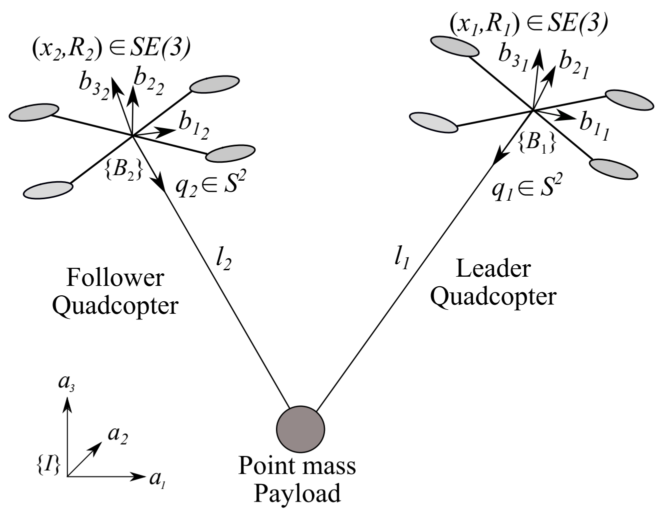

Fig. 1. Line diagram of the two quadcopters with a cable-suspended point mass payload system

quadcopters given as in Eq. (1), where $(\phi_i, \theta_i, \psi_i)$ represents roll, pitch and yaw angle of the quadcopter [18]. The angular position of each cable is measured from a_1 and a_2 axes denoted as ϕ_{p_i} and θ_{p_i} respectively which also can be represented in two sphere as $q_i = \{a \in \mathbb{R}^3 : ||a|| = 1\}$. Corresponding expression for q_i is given in Eq. (2). The angular velocity of the quadcopter is denoted as $\omega_i = [\omega_{1_i}, \omega_{2_i}, \omega_{3_i}]^T$.

$$R_i = \begin{bmatrix} c\psi_i c\theta_i - s\phi_i s\psi_i s\theta_i & -c\phi_i s\psi_i & c\psi_i s\theta_i + c\theta_i s\phi_i s\psi_i \\ s\psi_i c\theta_i + c\psi_i s\phi_i s\theta_i & c\phi_i c\psi_i & s\psi_i s\theta_i - c\theta_i s\phi_i c\psi_i \\ -s\theta_i c\phi_i & s\phi_i & c\phi_i c\theta_i \end{bmatrix} \tag{1}$$

$$q_i = \begin{bmatrix} c\theta_{p_i} & 0 & s\theta_{p_i} \\ s\phi_{p_i} s\theta_{p_i} & c\phi_{p_i} & -c\theta_{p_i} s\phi_{p_i} \\ -c\phi_{p_i} s\theta_{p_i} & s\phi_{p_i} & c\phi_{p_i} c\theta_{p_i} \end{bmatrix} \tag{2}$$

where, $c(\cdot) = cos(\cdot), s(\cdot) = sin(\cdot)$. Using Newton-Euler equations, the equation of motion for two quadcopter with cable-suspended point mass payload system considering the cable remains taut is given in Eq. (3 - 5) as derived in [19].

$$m_i \ddot{X}_i = f_i R_i a_3 - m_i g a_3 + T_i q_i \tag{3}$$

$$J_i \dot{\omega}_i + \omega_i \times J_i \omega_i = M_i \tag{4}$$

$$m_0 \ddot{X}_0 = -\sum T_i q_i - m_0 g a_3 \tag{5}$$

where, f_i, M_i is the thrust force and moment generated by i^{th} quadcopter respectively. T_i represents tension in the i^{th} cable.

2.1 Wrench Closure Workspace Analysis

This work considers a leader-follower scheme where one quadcopter is considered a leader quadcopter autonomously controlled to direct the system's motion. Another quadcopter, follower quadcopter, is commanded that it tracks desired trajectory of the payload while keeping the cable taut. As preliminary work, a wrench closure workspace (WCW) analysis is carried out for follower quadcopter (indicated as 2) when leader quadcopter (indicated as 1) is fixed at the origin. Further, static equilibrium configuration (SEC) is considered for WCW analysis. At SEC, follower quadcopter hovers at a particular point in space with zero linear and rotational velocities. From Eqs. (3-5), at SEC, the attitude of quadcopter, forces, and tension in each cable can be found using Eqs. (6-8).

$$f_1 R_1 a_3 = m_1 g a_3 - T_1 q_1 \tag{6}$$

$$f_2 R_2 a_3 = m_2 g a_3 - T_2 q_2 \tag{7}$$

$$T_1 q_1 + T_2 q_2 = -m_0 g a_3 \tag{8}$$

Using parameter values given in the Table 1, wrench closure workspace for follower quadcopter is shown in Fig. 2 (a) and corresponding cross-section view of WCW from YZ plane passing through origin O is shown in the Fig. 2 (b).

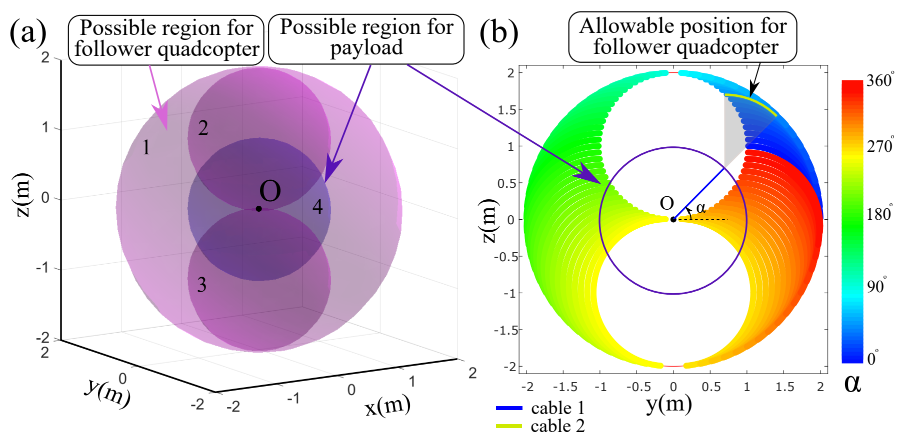

Fig. 2. (a) Static wrench closure workspace for follower quadcopter with respect to leader quadcopter (b) Cross-section view of WCW from YZ plane passing through origin O.

The volume contained by the magenta sphere 1 with the volume subtracted from the other two magenta spheres (2 & 3) in Fig. 2 (a) indicates wrench closure workspace for the follower quadcopter. WCW indicates the region in which the cable tension remains positive all the time. The surface of blue sphere 4 indicates a possible region for the payload position. As shown in Fig. 2 (b), an instance when cable 1 is at an angle of α from the Y axis is shown, and the highlighted arc shows the corresponding allowable position for follower quadcopter. Further, at every angular position of cable 1, i.e., from $0°$ to $360°$ corresponding allowable position for the follower quadcopter is shown by colored arcs. Using wrench closure workspace analysis following two corollaries are extracted at SEC.

1. If the translational position of the follower quadcopter is known with respect to leader quadcopter, then there exists only one feasible solution for the position of payload.
2. For the given angular position of the leader quadcopter's cable, we require only one parameter to define the system configuration completely. (This one parameter could be the translational position of the follower quadcopter or angular position of the follower quadcopter's cable.)

The trajectory for the follower quadcopter can be planned using the above corollaries to track the desired payload's trajectory. As the trajectory is planned based on the WCW, the resulting motion of the quadcopter ensures the positive tension in the cables.

2.2 Controller Design for follower quadcopter

Consider a case when the leader quadcopter is fixed at origin, and we want to track the desired trajectory for payload denoted as X_{p_d}. Using WCW analysis, we can define the corresponding trajectory for the follower quadcopter, which lets the payload track desired trajectory, X_{p_d}, while making sure the cables not get slacked. From the desired trajectory of the payload, cable 1's angular position can be found out, i.e., $q_{1_d} = X_{p_d}/l_1$. Using, WCW and corollary 2 if we know the angular position of the cable 2, i.e., q_{2_d} corresponding to q_{1_d}, we can calculate the desired position for follower quadcopter as given in Eq. (9).

$$X_{2_d} = l_1 q_{1_d} - l_2 q_{2_d} \tag{9}$$

To track the desired position for the follower quadcopter, a linear controller as described in [20] is used in this work. The follower quadcopter has to orient at a specific attitude and generate corresponding thrust force to balance its own weight and payload's weight at desired position X_{2_d}. Using Eqs. (6-8) the attitude of the quadcopter and thrust force at SEC can be found which is denoted as ϕ_{2_e}, θ_{2_e} and f_{2_e} respectively. The moment M_2 is calculated using PID controller as given in Eq. (10) and desired roll and pitch angle, i.e., ϕ_{2_d}, θ_{2_d} are calculated using Eqs. (11-12). In this work, the desired yaw angle is taken as zero, i.e., $\psi_{2_d} = 0$. The thrust force f_2 is calculated using Eq. (13) which controls the desired altitude z_{2_d}.

$$M_2 = \begin{bmatrix} k_P(\phi_{2_d} - \phi_2) + k_I \int(\phi_{2_d} - \phi_2)dt + k_D(\dot{\phi}_{2_d} - \dot{\phi}_2) \\ k_P(\theta_{2_d} - \theta_2) + k_I \int(\theta_{2_d} - \theta_2)dt + k_D(\dot{\theta}_{d_i} - \dot{\theta}_2) \\ k_P(\psi_{2_d} - \psi_2) + k_I \int(\psi_{2_d} - \psi_2)dt + k_D(\dot{\psi}_{2_d} - \dot{\psi}_2) \end{bmatrix} \tag{10}$$

$$\phi_{2_d} = \phi_{2_e} + \frac{-1}{g}(\ddot{y}_{2_d} - K_{\dot{y}_2}(\dot{y}_2 - \dot{y}_{2_d}) - K_{y_2}(y_2 - y_{2_d})) \tag{11}$$

$$\theta_{2_d} = \theta_{2_e} + \frac{1}{g}(\ddot{x}_{2_d} - K_{\dot{x}_2}(\dot{x}_2 - \dot{x}_{2_d}) - K_{x_2}(x_2 - x_{2_d})) \tag{12}$$

$$f_2 = f_{2_e} + m_2(\ddot{z}_{2_d} - K_{\dot{z}_2}(\dot{z}_2 - \dot{z}_{2_d}) - K_{z_i}(z_2 - z_{2_d})) \tag{13}$$

3 Results and Discussion

The performance of the proposed motion planning and control strategy is simulated using SIMSCAPE multibody dynamics software in SIMULINK. The parameters used to conduct simulations are listed in Table 1. Initially, both the cables are kept inclined at 45° from vertical plane. Hence, the angular position of the cables' are $q_{1_0} = [-0.707,\ 1,\ -0.707]^T$, $q_{2_0} = [0.707,\ 1,\ -0.707]$. For the initial three seconds, the system reaches to static equilibrium configuration from the defined initial configuration. After that, two different cases are considered for desired payload trajectory as given below.

1. Case - 1: Horizontal payload trajectory
   $x_{p_d}(t) = -0.5cos(\frac{t-3}{2})\,m$, $y_{p_d}(t) = 0.5sin(\frac{t-3}{2})\,m$, $z_{p_d}(t) = -0.866\,m$

2. Case - 2: Vertical payload trajectory
   $x_{p_d}(t) = -0.866\,m$, $y_{p_d}(t) = 0.5sin(\frac{t-3}{2})\,m$, $z_{p_d}(t) = -0.5cos(\frac{t-3}{2})\,m$

Description	Notation	Value
Mass of follower quadcopter	m_2	$1.2\,kg$
MOI of follower quadcopter	J_2	diag(0.016, 0.017, 0.032) kgm^2
Mass of payload	m_0	0.5 kg
Length of cables	l_1, l_2	$1\,m$

Table 1. Parameters used for simulation

Payload's desired and simulated translational position and follower quadcopter's desired and simulated translational position for tracking of $0.5\,m$ horizontal circular payload trajectory is shown in Fig. 3 and for tracking of $0.5\,m$ vertical circular trajectory is shown in Fig. 4. For the first three seconds, the system converges to SEC. After the third second, the follower quadcopter is commanded to track the desired trajectory X_{2_d} according to track the payload desired trajectory up to $30\,s$. Red plots show the desired trajectory to be tracked, and blue plots show simulated trajectories.

In case 1, root mean square (RMS) and standard deviation (STD) of error between desired and simulated trajectory for quadcopter's translational position are ($0.1514 \pm 0.1514\,m$, $0.1563 \pm 0.1559\,m$, $0.0461 \pm 0.0456\,m$) and for payload's translational position are ($0.0557 \pm 0.0556\,m$, $0.0565 \pm 0.0563\,m$, $0.0101 \pm$

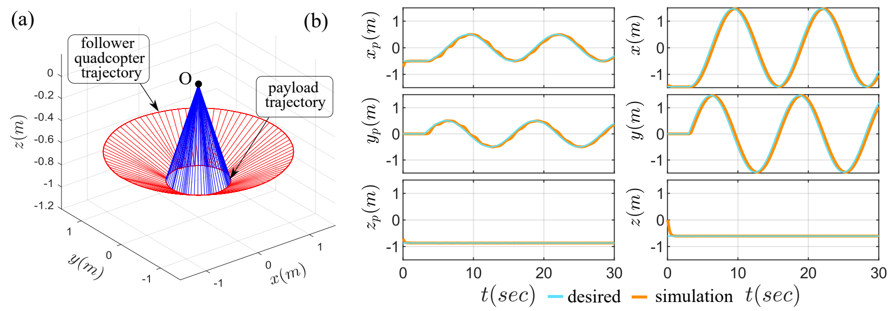

Fig. 3. Simulation results to track $0.5\,m$ horizontal payload trajectory (a) 3D plots for payload and follower quadcopter trajectory (b) Translational position of payload and follower quadcopter. Red plot shows the desired trajectory and blue plots show simulated trajectory

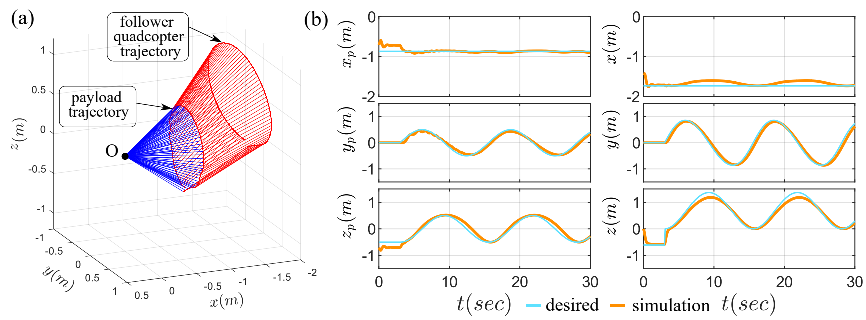

Fig. 4. Simulation results to track $0.5\,m$ vertical payload trajectory (b) translational position of payload and follower quadcopter. Red plot shows the desired trajectory and blue plots show simulated trajectory

$0.0101\,m$) along x, y, z axes respectively. In case 2, RMS and STD of error between desired and simulated trajectory for quadcopter's translational position are ($0.0870 \pm 0.0543\,m$, $0.0939 \pm 0.0899\,m$, $0.1231 \pm 0.1078\,m$) and for payload's translational position are ($0.0551 \pm 0.0543\,m$, $0.0649 \pm 0.0632\,m$, $0.1097 \pm 0.1032\,m$) along x, y, z axes respectively. The small values of the STD in payload and follower quadcopter indicates the presented controlled tracks the desired trajectory accurately.

The developed controller requires feedback of follower quadcopter's position with respect to leader quadcopter; by placing on-board cable attitude measurement devices as demonstrated in [21], the presented control strategy can be extended for the outdoor experiments easily. From the qualitative analysis, it is inferred that the motion of the follower quadcopter did not lead to accurate tracking of the desired position of the payload. It is because of oscillations of the payload while the transportation generates disturbances. However, as demonstrated in the literature [22, 23], the controller can be modified to incorporate minimization of the payload oscillations. Dynamic wrench closure workspace analysis will be carried out in future work from which agile transportation can be possible without slacking cables. Furthermore, outdoor experiments will be conducted to check the feasibility of the presented motion planning control strategy.

4 Conclusion

This work focuses on motion planning and control of two quadcopters with a point mass cable-suspended payload. The leader-follower scheme is considered to control the motion of the payload where one quadcopter is considered as leader quadcopter, which directs the motion of the system whereas another quadcopter, follower quadcopter, is commanded such that it tracks desired trajectory of the payload. Further, static wrench closure workspace analysis is carried out for follower quadcopter when leader quadcopter is fixed while helps to generate slackness-free trajectories for the follower quadcopter. The proposed motion planning modality is simulated using SIMSCAPE multibody dynamics software to check its performance. The simulation results demonstrated the feasibility of the proposed methodology in enabling slackness-free motion of the payload. The trajectory generation in this work is less computational costly.

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